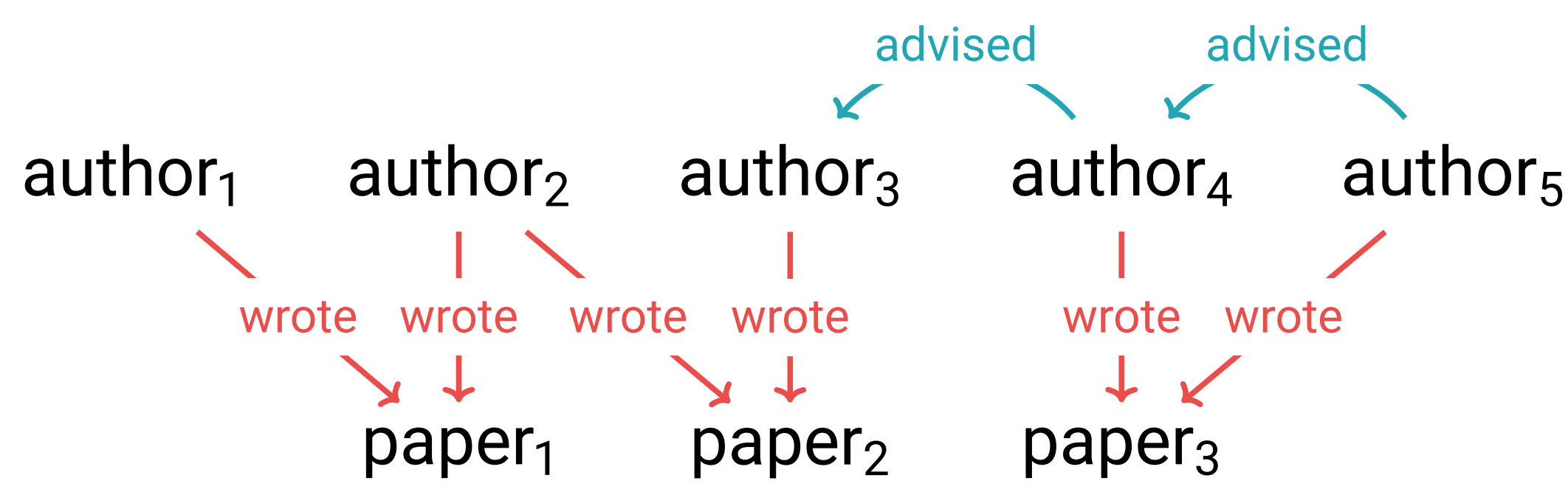


## Graph Databases

**Graph database:** an edge-labelled finite directed graph.



## Quest

Find expressive classes of queries with **fast** evaluation.

## Tree-width

**Tree-width:** Measure of how far a graph is from a tree.

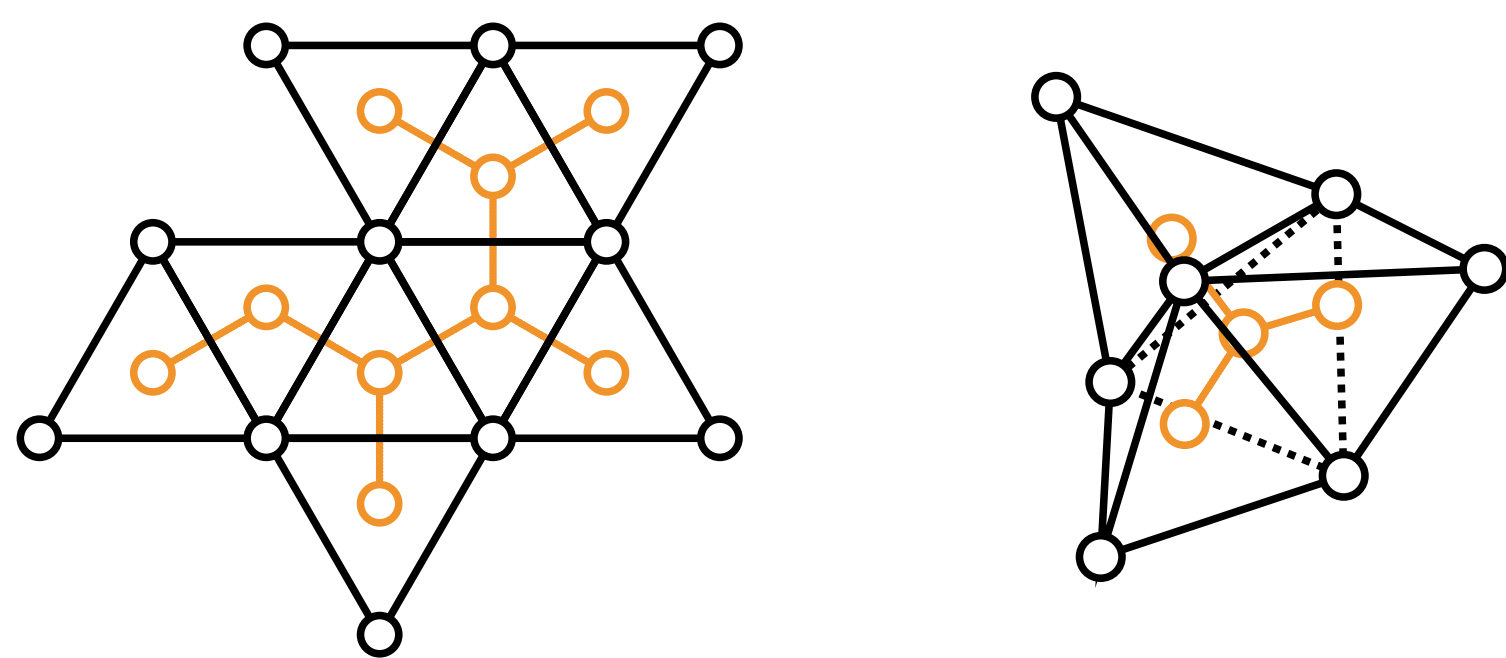
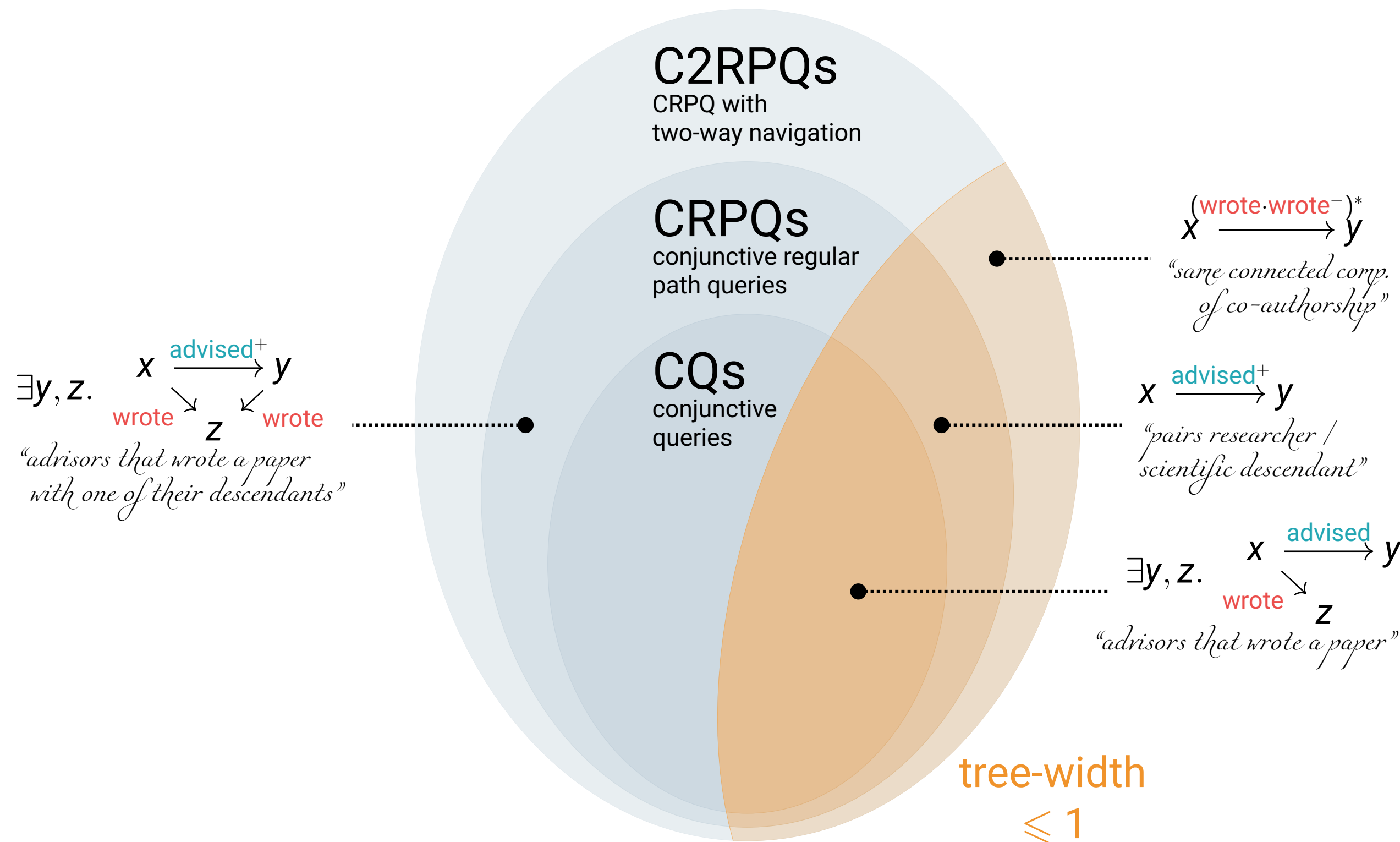


FIGURE: Graphs of tree-width  $k = 2$  (left-hand side) and  $k = 3$  (right-hand side). Each graph can be seen as an **underlying tree** whose nodes are replaced by a  $k$ -dimensional shape.

## Path Queries



**Proposition:** Evaluation of CQs/CRPQs/C2RPQs is **NP-complete**, but becomes **polynomial time** when restricted to queries of tree-width  $\leq k$  for each  $k \in \mathbb{N}$ .

**Corollary:** Graph theory can be useful (sometimes).

## Question

Can we decide if a query is equivalent to a query of tree-width at most  $k$ ?

## Semantic tree-width

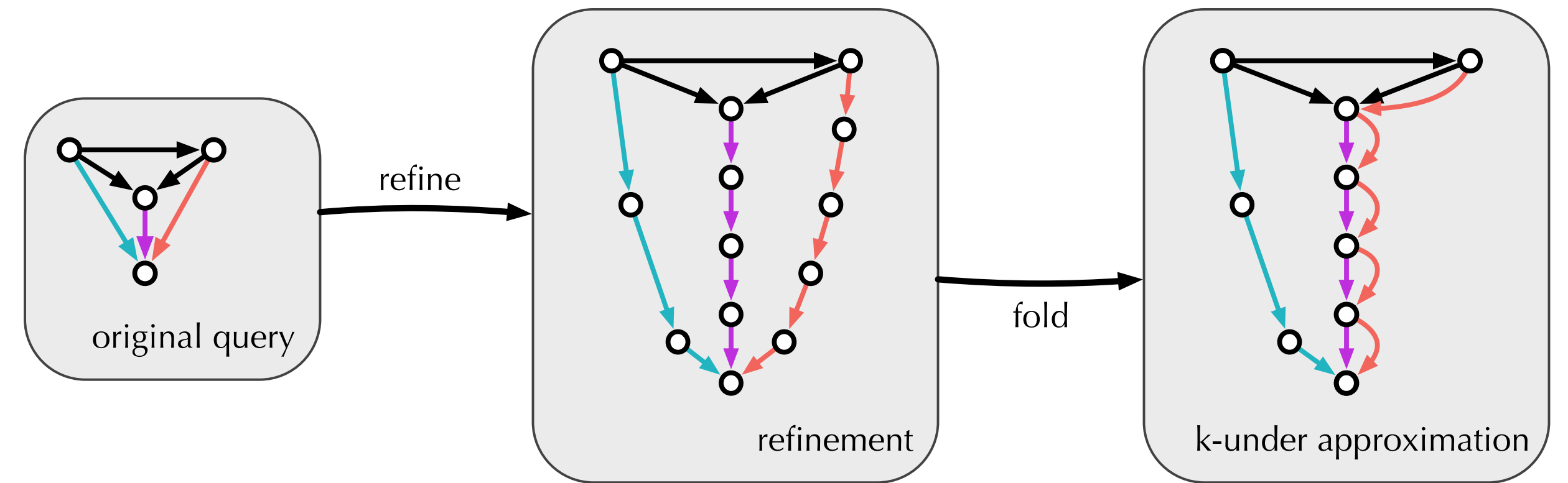
**Definition:** A C2RPQ has **semantic tree-width** at most  $k$  if it is equivalent to a finite union of C2RPQs of tree-width at most  $k$ .

**Theorem:** Semantic tree-width  $k$  is decidable and effective:

- ▶ if  $k = 1$  (EXPSpace-complete) [BBRV13]
- ▶ if  $k \geq 2$  (2EXPSpace and EXPSpace-hard) [FM23]

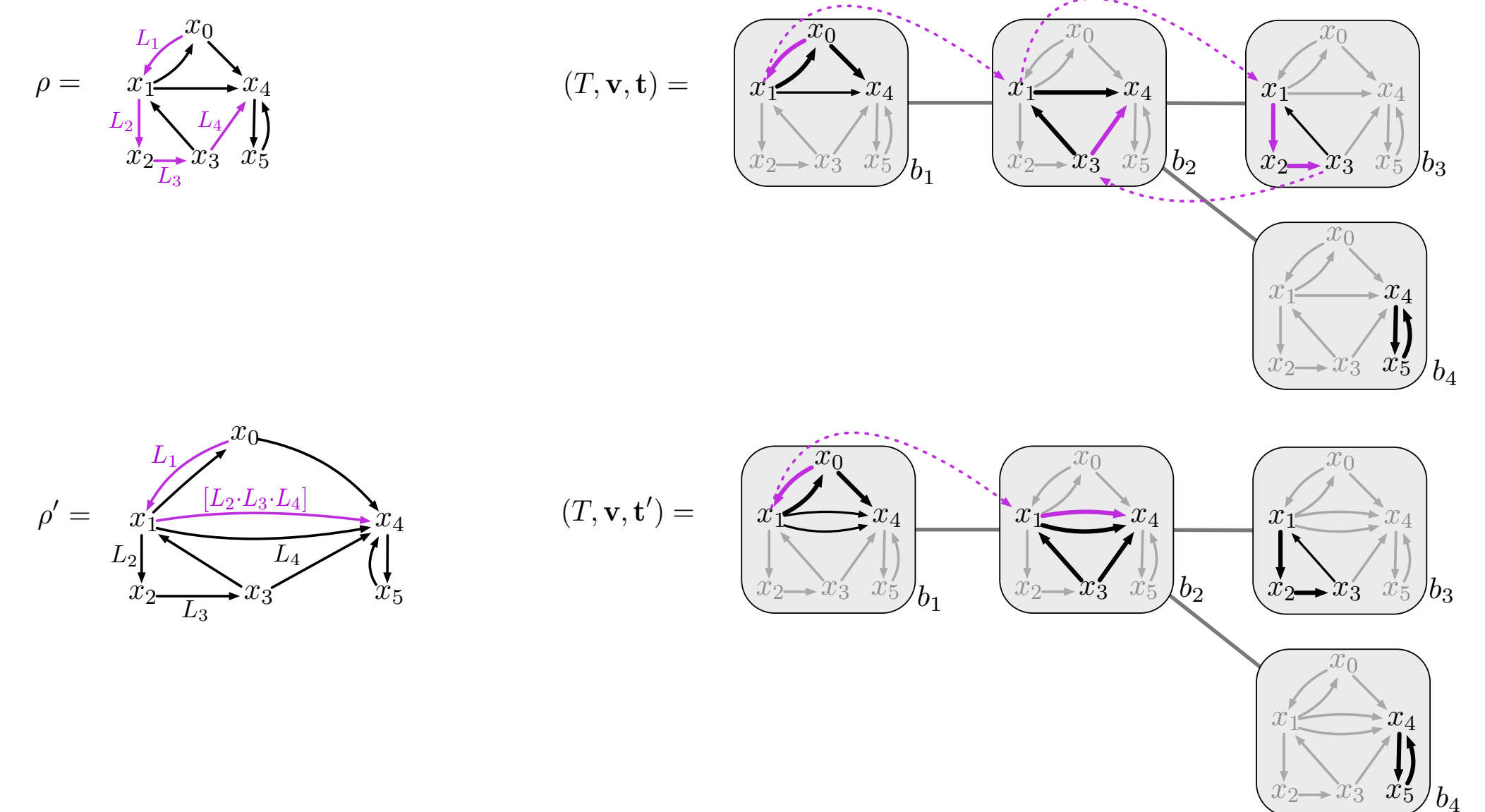
## Proof idea (by intimidation drawing)

A  **$k$ -under approximation** is any C2RPQ of  $\text{tw} \leq k$  obtained by:

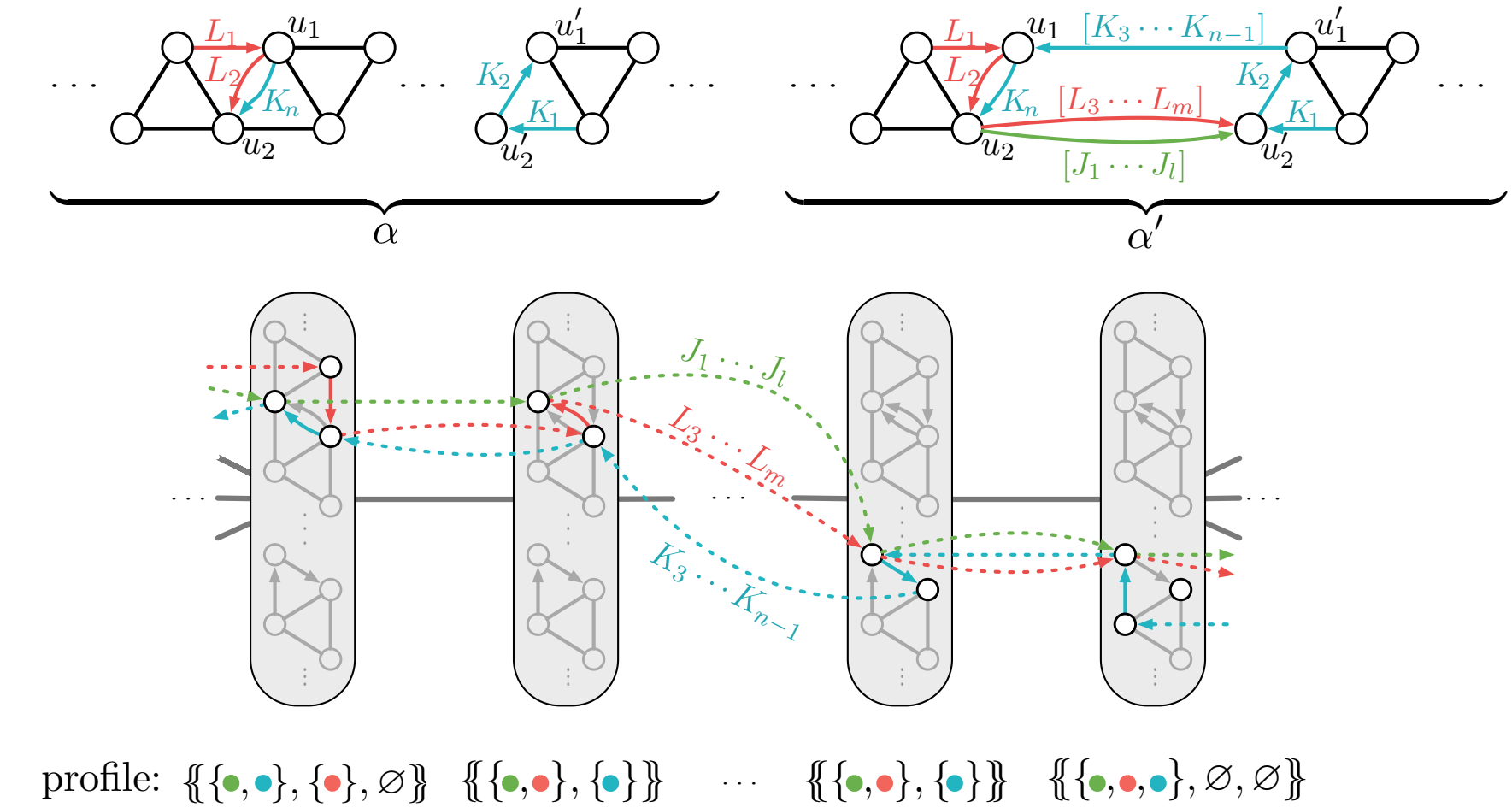


**Key lemma [FM23]:** The infinite set of  $k$ -under approximations is effectively expressible as a finite union of C2RPQs. *Proof:* Bound the size of refinements while preserving the semantic.

1. Make each induced path in the tree-decomposition acyclic:

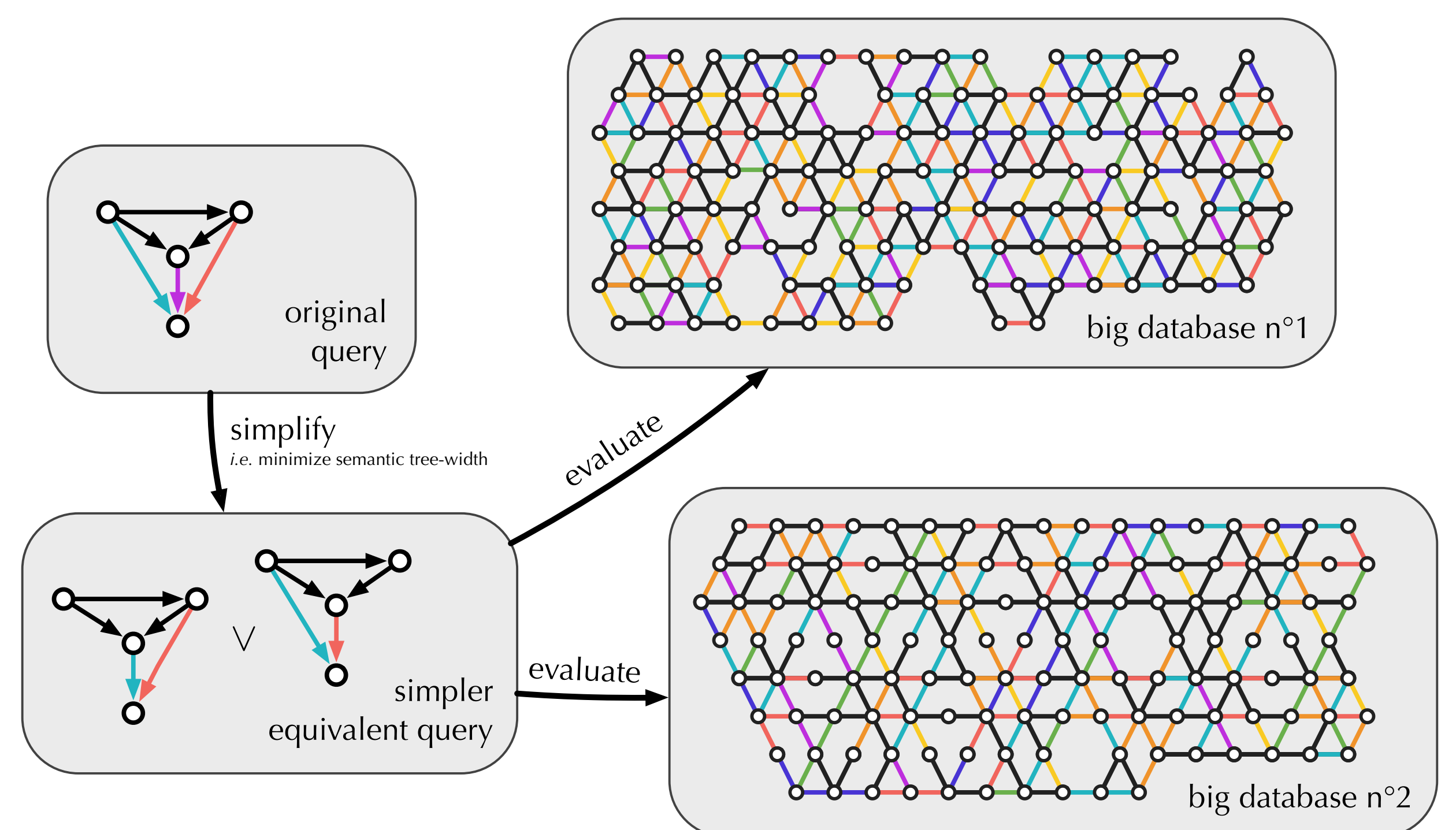


2. Bound the length of internal paths in the tree-decomposition:



## Evaluating queries of bounded semantic tree-width

**Corollary [FM23]:** C2RPQs of semantic tree-width at most  $k$  have **FPT evaluation** in the size of the query:  $\mathcal{O}(f(|\text{query}|) \cdot |\text{db}|^{k+1})$ .

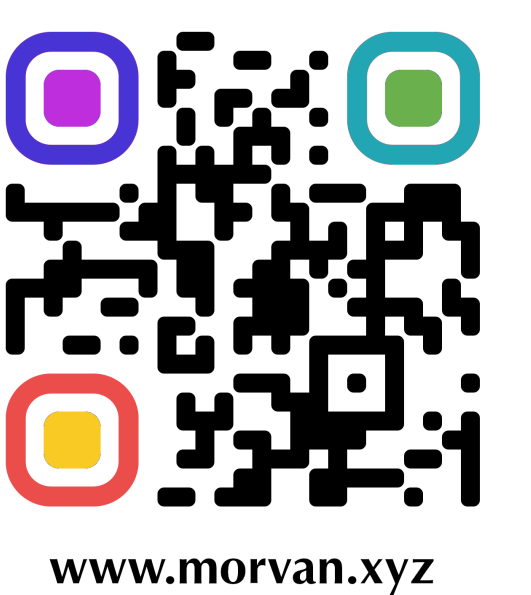


**Open question:** Does every recursively enumerable class of queries with FPT evaluation have bounded semantic tree-width?

**Grohe's theorem:** this is true for conjunctive queries [Gro03].

## References

- [BBRV13] Pablo Barceló Baeza, Miguel Romero, and Moshe Y Vardi. Semantic acyclicity on graph databases. In *PODS*, 2013.
- [FM23] Diego Figueira and Rémi Morvan. Approximation and Semantic Tree-Width of Conjunctive Regular Path Queries. In *ICDT*, 2023.
- [Gro03] Martin Grohe. The complexity of homomorphism and constraint satisfaction problems seen from the other side. In *FOCS*, 2003.



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\* This poster does not contain any internal hyperlink. This result was achieved by not using the *knowledge* package nor its companion tool *knowledge-clustering*.