

Quantitative algebraic characterisations on truly infinite words

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joint works with

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Highlights '22 - Paris

I ♥ algebra!

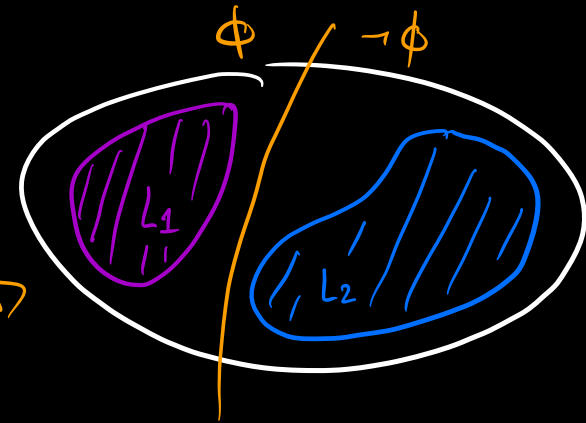
Thm [Schützenberger '65 & McNaughton-Papert '71]

regular lang. $\rightarrow L$ is definable in first-order logic
IFF
the syntactic monoid of L is aperiodic

Ex $(aa)^*$ not definable in FO

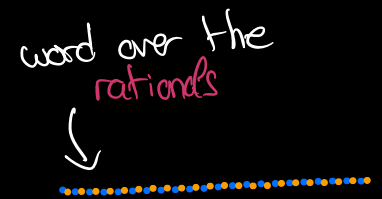
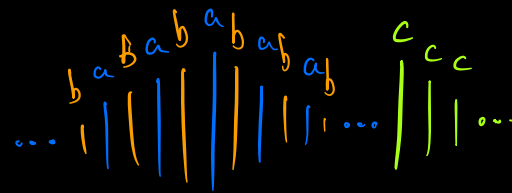
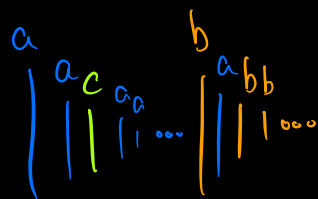
Thm [Hendickell '88]

L_1 and L_2 can be separated by first-order logic
IFF (but cute!)
some condition on a weird algebraic construction ...
(aperiodic 2-pointlike sets)



To infinity and beyond!

What about bigger words?

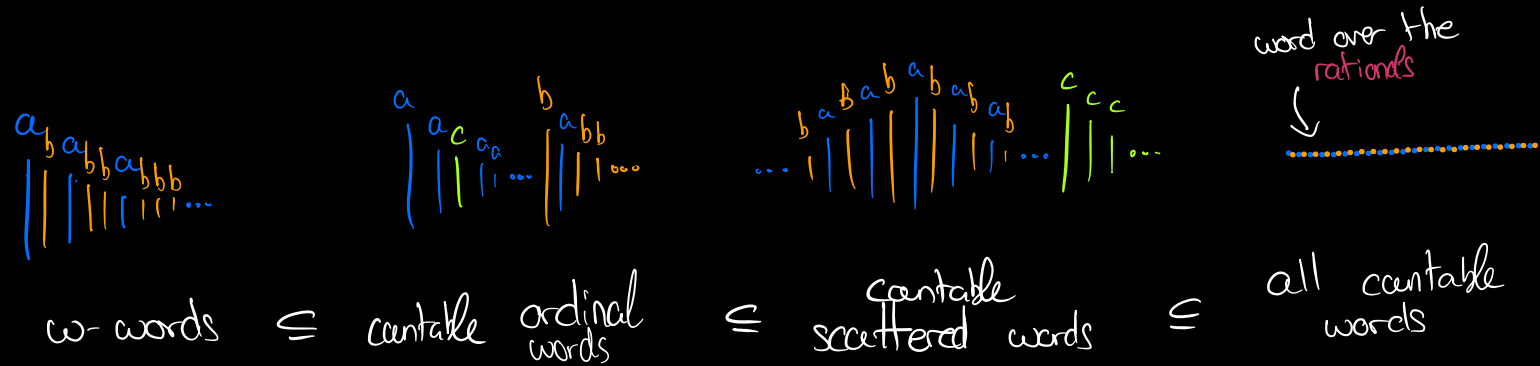


$$\omega\text{-words} \subseteq \text{countable ordinal words} \subseteq \text{countable scattered words} \subseteq \text{all countable words}$$

- Nice algebraic notions to study regular languages
- Nice algebraic characterisations of first-order definability!

Not a lot of results on separation... until now!

Qualitative & quantitative characterisations



Characterisation
of FO-definability

[Perin '84]

cannot
detect
groups

[Beton '01]

[Bis-Curton '11]

cannot
detect
groups and
gaps

[Colcombet -
Sreejith '18]

cannot
detect
groups, gaps
and shuffles

Characterisation
of FO-separability

[Place-Zitoun '16]

[Colcombet - van Gool -
Fossacs '22]

[Colcombet - M.
unpublished]

working on it...

+ We're also studying
WTSO, FO[cut], FO[ord], ...

